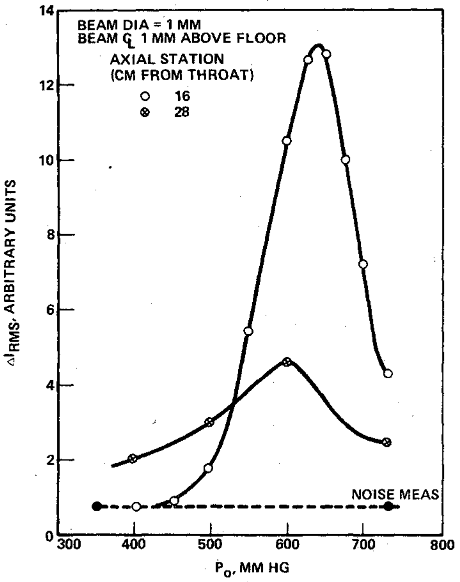

Fig. 4 Variation of peak rms intensity fluctuation with tunnel pressure.

fluctuation intensity, and therefore in ΔI_{rms}, is characteristic of transitional boundary-layer flow. In fact, the present results are remarkably similar to those obtained by Owen,[1] who used surface mounted hot film sensors to detect transition in supersonic boundary layers.

A comparison of the effect predicted by Eq. (3) and the experimental measurements is made using data obtained in the test section for fully-developed turbulence at a tunnel pressure $P_0 = 730$ mm Hg. For air, the variation in index of refraction Δn can be related to changes in density $\Delta\rho$ as follows

$$\Delta n = 0.000294\ \Delta\rho/\rho_0 = 0.00294\ (\Delta\rho/\rho)\ (\rho/\rho_0)$$

For $M_\infty = 3$ the freestream density, expressed in terms of the stagnation density (which in this case is nearly equal to the density at NTP), is $\rho_\infty = 0.076\ \rho_0$ while for the adiabatic case the density at the wall, as assuming constant static pressure across the boundary layer and a recovery factor near unity, is $\rho_w = \rho_0 p_w/p_0 = 0.027\ \rho_0$. Since the beam was located at about 0.3δ from the tunnel floor, we will use $\bar{\rho} \approx 0.05\rho_0$ and assume from Kistler's measurements[5] that the rms density fluctuation, normalized by the local mean, is 5%, yielding $\overline{(\Delta n)^2} \approx 5.5 \times 10^{-13}$. From Sutton's simplified scattering model,[6] the mean square angular fluctuation of a light ray traversing the turbulent boundary layer can be expressed as:

$$\overline{(\Delta\theta)^2} = \pi\overline{(\Delta n)^2}\ L/\Lambda$$

so that for out experiment

$$\overline{(\Delta x)^2} = \overline{(\Delta\theta)^2}\ \ell^2 = \pi\overline{(\Delta n)^2}\ L\ell^2/\Lambda$$

where L, the boundary-layer width = 7.5 cm, ℓ the optical path length = 24 m, and Λ, the turbulent scale length = $\delta/5 \simeq 0.1$ cm.[7] Substituting the appropriate values into the previous expression, we obtain

$$\left(\overline{(\Delta x)^2}\right)^{1/2} = \Delta x_{rms} \simeq 0.028 \text{ cm}$$

The parameter k is found by differentiating Eq. (3) with respect to $\bar{x}$ and setting $d(\Delta I_{rms}) = 0$. Thus $k = \sqrt{2}\bar{x}$, where $\bar{x}$ is the position for which ΔI_{rms} is a maximum. From the present tests this occurs at $\bar{x} = 0.6$ cm, so that $k = 0.84$ cm and from Eqs. (3) $I'\ (\equiv \Delta I_{rms}/I(\bar{x}))$ is 0.047 while from the experimental measurements $I' = 0.01$. The observed effect, therefore, is no more than a factor of 5 away from the prediction of Eq. (3). Of course, this discrepancy is symptomatic of the approximations in the analysis forced by incomplete knowledge of optical scattering[6] and of the details of supersonic turbulence. The important point is that the quantity I', whether equal to 0.01 or 0.05, is actually large enough to be measured simply even by unsophisticated laboratory set-ups. Another important point is that the actual magnitude of the sudden increase in I' (as shown on Fig. 4, for instance) at transition is not important to the detection of the transition point or region, an advantage common to other transition-detection methods as well.

Conclusions

We conclude from these results that a rather simple method of transition detection with a laser beam is available. The remote-sensing nature of the technique makes it attractive for use in ground tests, expecially if the beam can be made to pass through a width of the layer several times its thickness. The results are sufficiently encouraging to warrant an extension of the tests just described into the lower Mach number regime.

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Convergence of an Iterative Procedure for Large-Scale Static Analysis of Structural Components

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Introduction

CONDITIONS for convergence have been derived for an iterative procedure, proposed by Newman and Goldberg,[1] which is currently being employed to efficiently determine static stresses in the space-shuttle thermal protection system.[2] This procedure can also be used effectively for the analysis of other built-up component structures in which one of the substructures possesses a dominant stiffness which largely influences the deformations of the entire system. The general idealization being considered is represented in Fig. 1.

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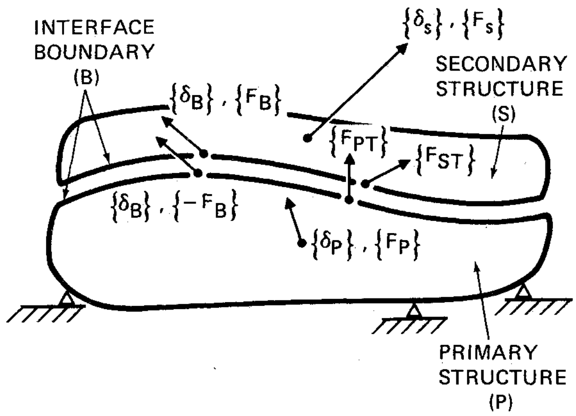

Notes:
1. $\{F_S\}$ and $\{F_P\}$ consist of both mechanical and equivalent thermal loads.
2. Interface boundary load $\{F_B\}$ is mechanical load only.
3. Thermal loads at interface boundary are $\{F_{ST}\}$ on secondary structure and $\{F_{PT}\}$ on primary structure.

Fig. 1 Representation of general structural idealization.

The dominant, relatively stiff, member is termed the primary structure. The less stiff secondary structure may be composed of one or more substructures that are not connected to one another but each of which is connected to the primary structure.

In Ref. 2, the primary structure consisted of the stiffened aluminum skin of the shuttle, and the secondary structure consisted of a low-strength covering of individual reusable surface insulation tiles. Because of the detailed structural grid required to determine three-dimensional stresses in each tile, computer solution by the conventional direct-stiffness or static-coupling methods would be too time consuming. Also, when the direct-stiffness method is used, combining components with diverse stiffness characteristics would result in numerical precision problems. These difficulties were successfully resolved using the iterative procedure described in Ref. 2 and briefly repeated here.This procedure is suitable for both mechanical and thermal static loads. An extension for free vibration problems has also been developed by Ojalvo.[3]

Iteration Procedure

The iteration procedure is initiated by estimating the deformation of the primary structure; for example, in the first step all mechanical loads may be assumed to be applied directly to the primary structure in the absence of the secondary structure and the approximate primary-structure deformation could be computed. In subsequent iteration steps, the jth iterate primary-structure deflections $\{\delta_B{}^{(j)}\}$ at the interface boundary are imposed upon the secondary structure and the boundary loads $\{F_B{}^{(j)}\}$ required to produce these deflections are computed. If the secondary structure consists of several independent members, this computation could be carried out individually for each member; thus smaller capacity programs could be used for the secondary members, and data storage requirements could be reduced. The iteration cycle is completed by applying the interface reaction $\{-F_B{}^{(j)}\}$ to the primary structure and computing its updated deflections $\{\delta_P{}^{(j+1)}\}$ which are then compared to the previous values for convergence. It will be shown that the mathematical condition for convergence of this procedure is that the maximum eigenvalue λ_{max} of the equation

$$[K_S]\{x\} = \lambda[K_P]\{x\} \tag{1}$$

must be less than 1, where $[K_S]$ is the stiffness matrix relating an increase in the interface boundary deformations on the secondary structure to the resulting increase in the boundary loads, and $[K_P]$ is the same matrix for the primary structure. These matrices will be formally defined in terms of the partitions of the secondary- and primary-structure stiffness matrices, respectively.

Proof of Convergence Criterion

Referring to Fig. 1, the load-deflection relation for the secondary structure is

$$\begin{bmatrix} K_{BB} & K_{BS} \\ K_{SB} & K_{SS} \end{bmatrix} \begin{Bmatrix} \delta_B^{(j)} \\ \delta_S^{(j)} \end{Bmatrix} = \begin{Bmatrix} F_B^{(j)} + F_{ST} \\ F_S \end{Bmatrix} \tag{2}$$

where the nomenclature indicated in Fig. 1 has been used and the superscript j indicates that the equation is being applied for the jth iteration. If $\{\delta_S{}^{(j)}\}$ is eliminated from Eq. (2), the following relationship for the interface load $\{F_B{}^{(j)}\}$ is obtained

$$\{F_B{}^{(j)}\} \equiv [K_S]\{\delta_B{}^{(j)}\} + \{F_S'\} \tag{3}$$

where

$$[K_S] = [K_{BB}] - [K_{BS}][K_{SS}]^{-1}[K_{SB}] \tag{4}$$

and

$$\{F_S'\} = [K_{BS}][K_{SS}]^{-1}\{F_S\} - \{F_{ST}\} \tag{5}$$

In the iterative procedure $\{F_B{}^{(j)}\}$ is obtained from Eq. (3),and $\{-F_B{}^{(j)}\}$ is applied to the primary structure to compute $\{\delta_B{}^{(j+1)}\}$. The appropriate relationship is derived from the following load-deflection equation for the primary structure

$$\begin{bmatrix} \bar{K}_{BB} & K_{BP} \\ K_{PB} & K_{PP} \end{bmatrix} \begin{Bmatrix} \delta_B^{(j+1)} \\ \delta_P^{(j+1)} \end{Bmatrix} = \begin{Bmatrix} -F_B^{(j)} + F_{PT} \\ F_P \end{Bmatrix} \tag{6}$$

Eliminating $\{\delta_P{}^{(j+1)}\}$ from Eq. (6) yields

$$\{\delta_B{}^{(j+1)}\} = -[K_P]^{-1}\left(\{F_B{}^{(j)}\} + \{F_P'\}\right) \tag{7}$$

where

$$[K_P] = [\bar{K}_{BB}] - [K_{BP}][K_{PP}]^{-1}[K_{PB}] \tag{8}$$

and

$$\{F_P'\} = [K_{BP}][K_{PP}]^{-1}\{F_P\} - \{F_{PT}\} \tag{9}$$

It has been assumed that the primary structure is statically supported so that $[K_P]$ is nonsingular. The iterative procedure is accomplished by repetitive application of Eq.s (3) and (7). An equivalent recursion relation is obtained by substituting Eq. (3) into Eq. (7); viz,

$$\{\delta_B{}^{(j+1)}\} = -[K_P]^{-1}[K_S]\{\delta_B{}^{(j)}\} - [K_P]^{-1}\left(\{F_S'\} + \{F_P'\}\right) \tag{10}$$

If the exact solution is obtained, the interface boundary deflection of the secondary structure, $\{\delta_B{}^{(j)}\}$ appearing in Eq. (3), will be equal to the corresponding deflection of the primary structure, $\{\delta_B{}^{(j+1)}\}$ in Eq. (7). Consequently, for the exact solution, Eq. (10) is identically satisfied when $\{\delta_B{}^{(j+1)}\} = \{\delta_B{}^{(j)}\} = \{\delta_B\}$; i.e.,

$$\{\delta_B\} = -[K_P]^{-1}[K_S]\{\delta_B\} - [K_P]^{-1}\left(\{F_S'\} + \{F_P'\}\right) \tag{11}$$

To determine how an error in the jth iteration will propagate, $\{\delta_B{}^{(j)}\}$ is written as follows

$$\{\delta_B{}^{(j)}\} = \{\delta_B\} + \{\epsilon^{(j)}\} \tag{12}$$

where the error $\{\epsilon^{(j)}\}$ is simply defined as the difference between the jth iteration and the exact solution. Substitution of Eq. (12) into Eq. (10), then using Eq. (11) in the result, yields

$$\{\delta_B{}^{(j+1)}\} = \{\delta_B\} - [K_P]^{-1}[K_S]\{\epsilon^{(j)}\} \qquad (13)$$

These manipulations are repeated; however Eq. (13) is used in place of Eq. (12). By continuing this process, the following equation is obtained

$$\{\delta_B{}^{(j+n)}\} = \{\delta_B\} + (-1)^n \left([K_P]^{-1}[K_S]\right)^n \{\epsilon^{(j)}\} \qquad (14)$$

Next, $[K_P]^{-1}$ $[K_S]$ is expressed in terms of its eigenvector matrix $[X]$ with columns $\{x_i\}$ and its eigenvalue matrix $[\Lambda]$ (which is diagonal with the eigenvalues λ_i on the diagonal)

$$[K_P]^{-1}[K_S] = [X]\,[\Lambda]\,[X]^{-1} \qquad (15)$$

Since

$$\left([K_P]^{-1}[K_S]\right)^n = [X]\,[\Lambda]^n[X]^{-1} \qquad (16)$$

and the eigenvalues of a positive-definite or semidefinite matrix must be non-negative, Eq. (14) shows that the iteration procedure will be convergent for all errors $\{\epsilon\}$ if and only if the magnitudes of all eigenvalues of $[K_P]^{-1}[K_S]$ are less than unity.

To obtain some physical insight into this convergence criterion, assume that deflections corresponding to any eigenvector, $\{x_i\}$, are applied to the secondary structure at the interface points and that these are the only points that are loaded. Equation (3) shows that the load on the secondary structure is $[K_S]\{x_i\}$. Next, suppose that an equal and opposite load is applied to the primary structure. Equation (7) shows that the resulting deflection is

$$[K_P]^{-1}[K_S]\{x_i\} = \lambda_i\{x_i\} \qquad (17)$$

Thus, the primary-structure deflection is λ_i times the imposed secondary-structure deflection $\{x_i\}$. Consequently, if $0 \le \lambda_i < 1$, the secondary-structure may be considered to be more flexible than the primary structure in the ith mode. Since the convergence criterion requires that $\lambda_{i\max} < 1$, the procedure will be convergent when the secondary structure is "more flexible at the interface boundary", so to speak, than the primary structure.

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On a Linear Time Varying System and Liapunov Criteria for Stability

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I. Introduction

A frictionless second-order system with time varying natural frequency is studied. For such a system analytical approaches exist in the literature dealing with the subject of missiles exiting or entering the atmosphere.[1] The case where the frequency of the system is reduced monotomically with time is the first example, known to the author, in which the solution diverges from its original position, but the system is stable in the sense of Liapunov.

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II. Analytical Approach[1]

Consider the linear time varying system

$$\ddot{x} + N(t)x = 0 \qquad (1)$$

where

$$N(t) \ge \epsilon > 0 \text{ for } t \ge 0 \text{ and } x \epsilon R^2$$

Multiply Eq. (1) by $2\dot{x}/N(t)$

$$2\ddot{x}\dot{x}/N(t) + 2x\dot{x} = 0 \qquad (2)$$

Integrate between t_1 and t_2 ($t_2 > t_1 >$ to)

$$\int_{t_1}^{t_2} \frac{2\ddot{x}\dot{x}}{N(t)}\,\mathrm{d}t + \int_{t_1}^{t_2} 2x\dot{x}t = 0 \qquad (3)$$

The first term in Eq. (3) is integrated by parts

$$\int_{t_1}^{t_2} \frac{\dot{N}(t)}{N^2(t)}\,\dot{x}^2\mathrm{d}t + \frac{\dot{x}^2}{N(t)}\Big|_{t_1}^{t_2} + x^2\Big|_{t_1}^{t_2} = 0 \qquad (4)$$

In accordance with the linear theory for constant coefficients assume that the behavior of $x(t)$ is of an oscillatory type, and choose t_1 and t_2 to be at two consecutive peaks, where at a peak:

$$\dot{x} = 0 \qquad (5)$$

So, Eq. (4) becomes

$$x^2(t_1) - x^2(t_2) = \int_{t_1}^{t_2} \frac{\dot{N}(t)\dot{x}^2}{N^2(t)}\,\mathrm{d}t \qquad (6)$$

From Eq. (6) it is obvious that if $\dot{N}(t) < 0$ then

$$x^2(t_1) < x^2(t_2) \qquad (7)$$

Which means that the envelop of $x(t)$ diverges. Similarly for $\dot{N}(t) > 0$ the envelop of $x(t)$ converges.

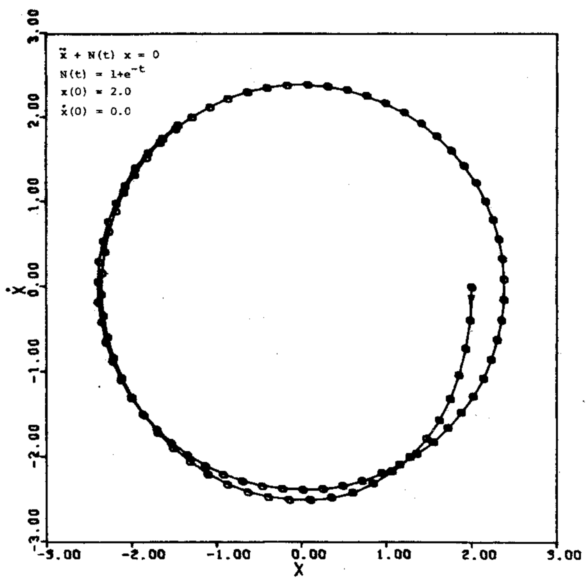

Fig. 1 A trajectory for the decreasing natural frequency case.